Department of Mathematics and Statistics

# Junior Mathematics Competition 2018 Solutions and Comments

**Web:** maths.otago.ac.nz/jmc

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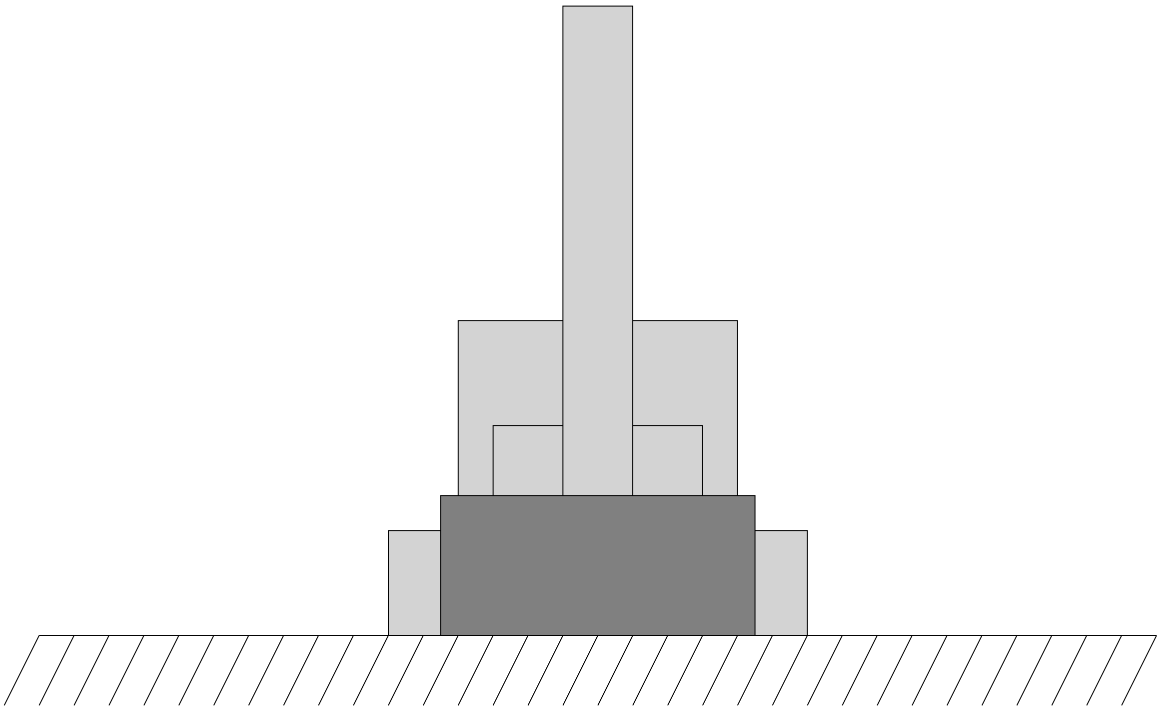

## Year 9 (Form 3) Prize Winners

| | | |
|---|---|---|
| **First** | Grace Wu | St Cuthbert's College |
| **Second** | James (He) Xu | Kristin School |
| **Third** | Michael Li | Macleans College |

## Top 30 (in School Order):

David Evans, Canterbury Home Educators
Samuel Darr, Hutt Intermediate School
Martin Brook, John McGlashan College
Wesley Ngo, Kings College
Catherine Zhang, Lynfield College
Wei Tian Teo, Macleans College
Regina Yun, Macleans College
Jessica Hong, Macleans College
Kerwin Maass, Newlands College
Henry Zaslow, Onslow College
Jessica Wang, Rangitoto College
Emma Ying, St Cuthbert's College
Braden Dye, St Kentigern College
Hamish Kerr, Hamilton Boys' High School
Rhiannon Mackie, Hutt Valley High School
Ranithu Sendiv Rodrigo, John Paul College
Andrew Yu, Kings College
Kevin Lin, Macleans College
Hosea Tong-Ho, Macleans College
Nancy Chen, Macleans College
James Hui, Macleans College
Ziyue (Claire) Xu, Northcross Intermediate
Lucy Aitken, Rangi Ruru Girls' School
Grace Pui, St Cuthbert's College
Brena Merz, St Cuthbert's College
Max Feng, St Kentigern College
Tony Shi, Westlake Boys' High School

## Year 10 (Form 4) Prize Winners

| | | |
|---|---|---|
| **First** | Michael Kennedy | Liston College |
| **Second** | Oliver Dai | Macleans College |
| **Third** | James Sutton | Rongotai College |

## Top 30 (in School Order):

Aldous Schollum, Avondale College
Duncan Hamilton, Christchurch Boys' High School
Tobias Devereux, Kavanagh College
Eric Zhang, Kristin School
Maggie Chen, Macleans College
Lisa Sandbrook, Macleans College
Anna Li, Orewa College
Kiki Su, Pakuranga College
Tony Schaufelberger, Rutherford College
Arisa Mori, St Andrew's College
Justin Xiang, St Kentigern College
Michelle Guan, St Kentigern College
Haokun He, St Kentigern College
Shreya Sriram, Avondale College
Harry Neil, Hillcrest High School
Jason Lee, Kings College
Annie Peng, Kristin School
Rick Han, Macleans College
Jeremy Huang, Macleans College
Oliver Lin, Pakuranga College
Seohyun Kim, Rangitoto College
Jet Shand-Pease, Selwyn College
Jenny Yuan, St Cuthbert's College
Nathan Xu, St Kentigern College
Evan Metcalfe, St Kentigern College
Phillip Han, Westlake Boys' High School
Taewon Yun, Westlake Boys' High School

## Year 11 (Form 5) Prize Winners

| | | |
|---|---|---|
| **First Equal** | Luke Ying Feng Bao | Auckland Grammar School |
| **First Equal** | Chen Huan Liu | Epsom Girls' Grammar School |
| **Third** | Terry Shen | Macleans College |

## Top 30 (in School Order):

Andrew Jia An Che, Auckland Grammar School
Mackinley HE, Auckland Grammar School
Isaac Hao-En YUAN, Auckland International College
Victoria Sun, Epsom Girls' Grammar School
Emily Zou, Glendowie College
Nathaniel Masfen-Yan, Kings College
Ranudi Lelwala, Macleans College
Yunge Yu, Macleans College
Darsh Chaudhari, Macleans College
Ray Wu, Newlands College
Yuhan (Linda) Tang, Pinehurst School
Xavier Yin, St Kentigern College
Grace Chang, St Kentigern College
Nathan Chen, Auckland Grammar School
Andrew Yu An Chen, Auckland International College
Max Young, Christchurch Boys' High School
Daniel Clark, Glendowie College
Ishan Nath, John Paul College
Hanbo Xie, Kings College
Jamie Wu, Macleans College
Angela Yang, Macleans College
Kelvin Gu, Macleans College
Jeremy Ishi, Pakuranga College
Minju Kim, Rangitoto College
Tony Yu, St Kentigern College
James Alexander, St Peter's College (Epsom)
Steven Wang, St Peter's School (Cambridge)

# 2018 Answers

As always only one method is shown. Often several methods exist. We don't guarantee that the method shown is the best or fastest.

**Question 1: 10 marks (Years 9 and below only)**

(a) A rectangular wall measures 4.5 m by 3 m. Show that the area of the wall is 13.5 $m^2$. **Either 4.5 $\times$ 3 or 3 $\times$ 4.5. An easy start but surprisingly a few students missed it out!**

(b) Suppose the area of a roll of wallpaper has dimensions 5 m by 0.5 m.

  (i) What is the area covered by one roll of wallpaper? **5 $\times$ 0.5 = 2.5 $m^2$. Well done. (Many students wrote 2.5 m instead of 2.5 $m^2$.)**

  (ii) How many 'whole' rolls of wallpaper would you need to cover the wall in part (a)? Your answer should not have a fraction or decimal component in it. **13.5 / 2.5 = 5.4 so 6 rolls are needed. Many students rounded 'down' to 5, leaving part of the wall bare of wallpaper. Remember that is generally better to buy a little too much than not enough!**

(c) If a window covers 2.7 $m^2$ on the wall, what percentage of the wall is covered by the window? **This was well answered in the main. 2.7 / 13.5 $\times$ 100 = 20%.**

**Question 2: 10 marks (Years 10 and below only)**

A store sells refrigerators with a **recommended** price of $1600. However they know that they cannot sell any at this price so they offer them at a 'special' price of $1200.

(a) What is the percentage discount of $1200 compared to $1600? **1200 / 1600 = 75% so the answer is a 25% discount. Reasonable, although a few students divided 1600 by 1200 and some left 75% as their answer.**

(b) In a Boxing Day Sale they offer the fridge at a special '20% saving' i.e. 20% off the **recommended** price of $1600.

  (i) What is the special Boxing Day 'Sale' price? **$1600 - $320 = $1280. Well answered in the main. The most common mistake was to leave $320 as the final answer.**

  (ii) Does this Sale Price represent a saving on the 'special' price or an increase? You don't have to show working. **It's an increase since $1280 is more than $1200. Generally well answered although students who wrote 'Yes' or 'No' weren't giving an answer at all.**

(c) They offer a 'further 20%' off the **Sale** Price if a customer pays cash. What percentage saving do these two percentages combined represent? **(1 - 0.8 $\times$ 0.8) = 36%. This was the first 'difficult' question. If we take $100 as the normal price then a 20% discount leads to an $80 sale price. 20% of $80 is $16 so the final price is $64, which represents a 36% discount. Some students worked it out. Generally this question was a good indicator as to whether a student eventually gained a Merit (or above) or not.**

(d) Another store usually makes 400% profit on items they sell. For example an item costing the store $20 would sell for $100. In a sale they offer the $100 item for 50% off the usual price. What percentage profit on the item do they still make? **50% of $100 is $50; thus a $30 profit still exists. If the original item cost the store $20 then they still make a profit of 30 / 20 = 150%. Many students worked this out, but it was also often wrong. 250% and 30% (or $30) were not uncommon. Non-sensible answers bigger than the original 400% occasionally appeared.**

**Question 3: 20 marks (All Years)**

**This question was fairly 'easy' for many students. Full credit was common.**

(a) Harry and Meghan plan to bake muffins for afternoon tea. However, the original recipe serves five people. They will have to adjust the amount of ingredients they need to serve the two of them.

(i) The original recipe calls for 100 g of butter. How much butter is needed for two people? **2/5 of 100 is 40 g. Ratio was well understood.**

(ii) The original recipe calls for 15/16 of a cup of blueberries. What fraction of a cup do they need for two people? Briefly show your working. $\frac{15/16}{5/2}$ **= 3/8. There were several candidates who reached 6/6.4 here. They showed no understanding of dealing with fractions.**

(iii) Harry and Meghan work out that they now need two cups of flour. How many cups of flour did the original recipe call for? **5 cups, since 2/5 of 5 is 2.**

(iv) They work out that for two people they need half a cup of plain yoghurt. How much plain yoghurt did the original recipe call for (i.e. for five people)? **0.5 × 5/2 = 1.25 cups. A few students got 5/4 but lost credit when they gave the final answer as 1.5 cups. 2.5 was also a 'common' mistake.**

(b) A good rule of thumb for converting ounces (oz) to grams (g) (although it is not perfect mathematically) is to use the formula 1 oz = 30 g. Use this to answer the following true or false questions (you don't need to show working):

(i) 2 g = 60 oz. **False. This proved to be the hardest part of the question! Several students didn't notice that it was 'back-to-front'.**

(ii) 1.6 oz = 48 g. **True. Well answered.**

(iii) 1 kg = 33 oz (to the nearest whole number). **True. (1000 / 30 = 33 to the nearest whole number.) Unfortunately this question is ambiguous and 'False' could be given by students who 'started at the right'. (33 × 30 = 990, not 1000.) The trouble with ambiguous questions is that a few students 'waste time' figuring which is the answer we looked for. A few students gave both answers, with explanations. The answer schedule was adjusted for the error.**

(c) There are 16 ounces (oz) in a pound. Remember that we are using the approximation 1 oz = 30 g.

(i) How many grams are there in 1 pound? **30 × 16 = 480 g. Almost universally answered correctly.**

(ii) How many pounds are there in a 6 kg of flour? **We were looking for 6000 / 30 = 200 (ounces) and then 200 / 16 = 12.5 (pounds). But most students gave 6000 / 480 = 12.5 which was fine. The problem with using one of your own answers in a later part of the question is that it may be wrong, and so your second answer becomes 'wrong' too. Fortunately '480' was extremely common and so 12.5 was also common.**

**Question 4: 20 marks (All Years)**

The ancient Egyptians only used two types of fractions, 2/3 and the unit fractions (fractions with 1 as the numerator), 1/2, 1/3, 1/4, 1/5 etc. They could write any fraction as additions of unit fractions:

e.g. 3/4 = 1/2 + 1/4 and 4/5 = 1/2 + 1/4 + 1/20 + 1/80.

(a) True or false? (You don't need to show working.) **These were generally well done, although some students couldn't handle parts (iii) and/or (iv).**

(i) 2/3 = 1/2 + 1/6 **True.**

(ii) 1/2 × 1/6 = 1/2 + 1/6 **False.**

(iii) 2/3 of 1/5 = 1/10 + 1/30 **True.**

(iv) $\frac{1/5}{1/2} = 1/10$ **False. Dividing fractions by other fractions caused many students difficulty.**

(b) (i) Write 3/24 as a unit fraction. **1/8. This question caused problems! Some students didn't realise that only one fraction was needed and they wrote things like 1/12 + 1/24.**

(ii) Write 3/5 as the sum of exactly two **different** unit fractions. **1/2 + 1/10. It was about here that the 'top' candidates started to separate themselves out from the 'bottom' candidates. Some couldn't work out the rest of the question but some just wrote the answers straight down.**

(c) Divide 11 loaves of bread equally between 12 people, so that each person receives the same amount. Then give your answer as the sum of three **different** unit fractions. **There were two possibile answers: 1/2 + 1/3 + 1/12 or 1/2 + 1/4 + 1/6. There was no credit for writing 11/12 on its own. A few wrote 12/11 for no credit either.**

(d) Consider the fraction 13/11. Write this as the sum of 2/3 and three unit fractions. In this question your unit fractions do not have to be all different. **The most 'common' answer that we expected was 2/3 + 1/3 + 1/11 + 1/11. However alternatives exist and many were sighted. (We used a computer to find what we think is the full set.)**

**The other possibilites are 2/3 plus any of the following:**

| | | |
|---|---|---|
| **1/3 + 1/6 + 1/66** | **1/2 + 1/67 + 1/4422** | **1/2 + 1/69 + 1/1518** |
| **1/2 + 1/99 + 1/198** | **1/2 + 1/102 + 1/187** | **1/2 + 1/70 + 1/1155** |
| **1/2 + 1/110 + 1/165** | **1/2 + 1/88 + 1/264** | **1/4 + 1/4 + 1/66** |
| **1/2 + 1/84 + 1/308** | **1/2 + 1/78 + 1/429** | **1/2 + 1/132 + 1/132** |
| **1/2 + 1/77 + 1/462** | **1/2 + 1/75 + 1/550** | **1/2 + 1/68 + 1/2244** |
| | **1/2 + 1/72 + 1/792** | |

**Question 5: 20 marks (All Years)**

A factorial is one of the 'building blocks' for many areas of Mathematics. For any whole number $n$, factorial $n$ (denoted by $n!$) is defined as

$$n! = n \times (n-1) \times (n-2) \ldots \times 3 \times 2 \times 1.$$

For example, the number $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

(a) Find the values of the following:

(i) 4! **24. Well answered in the main.**

(ii) (3 + 3)! **6! = 720. Well answered, although many tried to tell us that 3 + 3 = 9.**

(iii) 10!/8! **90. The easiest way to work this out is to notice that most terms divide out, and this leaves just 10 $\times$ 9 = 90. Some students 'expanded' top and bottom to reach horrendously large numbers which many couldn't handle.**

(iv) 16!/(14! $\times$ 2!) **120. This also simplifies to (16 $\times$ 15) / 2. One of the dangers of doing a full expansion is that you can produce rounding errors. Several students had fractional components to their answer; such answers received no credit.**

(b) Write the following expressions as simply as possible:

(i) $n!/(n-1)!$ ***n*. Mixed results. The 'proper' way is to note that every term except *n* itself divides out. Some tested values of *n* (like 3 or 4) to work the answer out. This approach worked, although hardly anybody seemed to be able to use such an approach in (ii).**

(ii) $n!/[(n-2)! \times 2!]$ $\boldsymbol{n(n-1)/2}$**. Since** $\boldsymbol{n! = n \times (n-1) \times (n-2)!}$ **the** $\boldsymbol{(n-2)!}$ **top and bottom will divide out, leaving *n*(*n* - 1) in the numerator and the (untouched) 2 in the denominator.**

A rowing coach has to select 2 rowers for a pairs competition out of 5 contenders. She decides to select them at random. This can be done in 10 ways. If the 5 rowers are Aroha, Bao, Charlotte, Debbie, and Emere, then she can pick any pair from this set:

$$\{AB, AC, AD, AE, BC, BD, BE, CD, CE, DE\}$$

The number 10 can also be found using $5!/(3! \times 2!)$.

(c) In how many ways can an athletics coach pick a team of 4 athletes at random from 7 choices?

$$\frac{\mathbf{7!}}{(\mathbf{4!} \times \mathbf{3!})} = \frac{\mathbf{7 \times 6 \times 5}}{\mathbf{6}} = \mathbf{35.}$$

**Some students tried to list all the possibilities. Nobody reached the answer using this method.**

(d) In 'Litto', you win a prize in the first division by correctly picking 4 different numbers out of 15. What are your chances of winning a first division prize if you have one entry of different numbers selected at random?

$$\frac{\mathbf{15!}}{(\mathbf{11!} \times \mathbf{4!})} = \frac{\mathbf{15 \times 14 \times 13 \times 12}}{\mathbf{24}} = \mathbf{1365.}$$

**This means there is one chance in 1365 if you pick numbers entirely 'at random' in 'Litto'.**

**Question 6: 20 marks (All Years)**

(a) Find a set of ten consecutive numbers between 100 and 199 (inclusive), such that:

- Exactly one number is a perfect square.
- Exactly one number is prime.
- The other eight numbers have at least one of 2, 3, or 5 as a factor.

Explain your reasoning for the set you give. (In other words state the perfect square, the prime, and show the other numbers have one of 2, 3, or 5 as a factor.) **There are four possible answers:**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Set A:** | **120** | ***121*** | **122** | **123** | **124** | **125** | **126** | **[127]** | **128** | **129** |
| **Set B:** | ***121*** | **122** | **123** | **124** | **125** | **126** | **[127]** | **128** | **129** | **130** |
| **Set C:** | **168** | ***169*** | **170** | **171** | **172** | **[173]** | **174** | **175** | **176** | **177** |
| **Set D:** | ***169*** | **170** | **171** | **172** | **[173]** | **174** | **175** | **176** | **177** | **178** |

**(Where a number in italics is a perfect square and a boxed number is a prime. In sets A and B 123 and 129 are multiples of 3 and 125 is a multiple of 5, while in sets C and D 171 and 177 are multiples of 3 and 175 is a multiple of 5. It should be clear that all numbers not yet mentioned are at the very least even.)**

**A few students gave sets of 11 numbers for partial credit. Several students appeared not to notice (or even know the meaning of?) the word 'consecutive'. Sets of ten numbers containing any number scattered between 100 and 200 appeared too often. Moreover 100 was often 'missed out' as a valid perfect square. The word 'inclusive' was also not universally known. We shall continue to use 'large' words towards the end of the competition. (We do try to 'restrain' ourselves towards the start.)**

(b) Is the set of numbers you found in (a) the only such set of numbers that can be found? If not, give another set of numbers with explanations. If so, explain why there are no other sets that satisfy the requirements. **Here you needed to explicitly state that there was another set apart from the one you gave in (a). Then you had to give (and explain) your second set.**

**Question 7: 10 marks (Years 10 and 11 only)**

Wiremu, a farmer, decides to put some lambs for two hours in a pen, one side of which is a very long brick wall. He will make the other sides from a long length of fencing. He doesn't need fencing for the side made from the brick wall. He decides that the pen will be 36 $m^2$ in area.

(a) If he makes a rectangular pen 2 m by 18 m (with the 18 m side being parallel to the brick wall), how much fencing will he use? **2 + 18 + 2 = 22 m. Well answered by those who reached the question.**

(b) If he does decide to make a rectangular pen with each side being a whole number of metres, find the minimum amount of fencing that he uses. Show working.

**The minimum is 17 m. To see this, consider the five integer factor 'pairs' of 36: (1, 36), (2, 18), (3, 12), (4, 9), and (6, 6). Thus the possible choices of fencing include:**

| | |
|---|---|
| **1 m by 36 m:** | **uses 38 m** |
| **2 m by 18 m:** | **uses 22 m** |
| **3 m by 12 m:** | **uses 18 m** |
| **4 m by 9 m:** | **uses 17 m** |
| **6 m by 6 m:** | **uses 18 m** |

**Of course there are more. For example '4 by 9' could be '9 by 4' and this therefore will become '9 + 9 + 4 = 22 m' instead of '4 + 4 + 9 = 17 m'. It depends on your orientation. But using the larger factor 'first' never produces a result smaller than 17 m.**
**For full credit you had to list several examples and draw the correct conclusion. About a dozen students used calculus. Only a few covered every aspect of the question. Full credit was possible. Such students have already become used to the idea that if they see the word 'minimum' they must use calculus. However other methods exist for many problems. Sometimes brute force is best, especially if there are fewer than 10 options!**

(c) Write 36 $m^2$ in hectares. **0.0036 ha. Several students had absolutely no idea what a hectare is: 100 m $\times$ 100 m = 10 000 $m^2$. It's about the size of two rugby fields side by side. (The prefix 'hecto-' means hundred.)**

**Question 8: 10 marks (Year 11 only)**

Wiremu decides to make the pen an equilateral triangle shape with all three sides being equal in length and one side along the long brick wall. He will need to fence the other two sides and he decides that the area of the pen will still be $36\,m^2$. Find how much fencing he will use. (Note: the final answer is not a whole number. Give your answer to one decimal place.)

**Let one side of the triangle have length $x$ metres, and let $A$ be the area of the pen ($36\,m^2$). Then we can use the formula for the area of triangle, $A = \frac{1}{2}ab\sin\alpha$, setting $a = b = x$ and $\alpha = 60°$.**
**So**

$$\begin{aligned} \frac{1}{2} \times x \times x \sin 60° &= 36 \\ x^2 &= 36 \times \frac{4}{\sqrt{3}} \text{ (or 83.138 to 3 decimal places)} \\ x &= 9.12 \end{aligned}$$

**So he uses $2x$ = 18.2 m (1 d.p.) of fencing. A few students multiplied $x$ by 3 to produce their final answer, forgetting that one side of the triangle used the brick wall.**

**This Year's Hints!**

These are some of the things we noticed this year.

(a) It is probably a good idea not to use an eraser. If you write something that you want to change simply put a *single* line through it and then write your new answer (and working if you feel it's needed). For the most part we ignore crossed-out work. But if you don't have time to replace it we will then consider said crossed-out material and if it is worth credit we will award the credit. (Sometimes crossed-out answers are actually correct!)

If you do use an eraser to the extent that we can't read the material you had written (or if you blacken it out completely) then we cannot give you any credit for work which might have been correct.

(b) Too many 'good' students are writing too much in the early questions then running out of time. It is probably a good idea to write the answer only (with perhaps one line, and only one, of working) and then coming back to the question at the end if you want to and have time.

Generally speaking, the later you are in a question, the more likely working is needed. Earlier questions should not need much working, but later questions likely will. There are always exceptions to this though; determining when to show working is a skill well worth learning.

This year we saw several Year 9 students gain full credit for Questions 1, 2 and 3, but they used so much space and time that there was nothing else answered. These students ended with a mark of 40, which was well short of a 'Merit', yet everything they wrote was correct!

(c) One marker would like to stress that the number 119 is NOT prime. It is in fact equal to $7 \times 17$. This number stopped many candidates from producing good results in Question 6.

It is very easy to check this if you have a calculator. Students without one are at a disadvantage. Certainly they can work most arithmetic problems out, but it uses up valuable time. Even a cheap 'four operations' calculator is better than no calculator at all.

Schools may have sets of old calculators which they can lend you for the competition.

(d) Don't write the question numbers down the left before you begin to write any answers. This is very important if you have only allowed one line for each answer. Answering Maths questions at secondary school often takes more than one line. Students who try to write a lot in one line often end up writing so small that we can't read their answer at all. Result? No credit.

(e) Don't waste time writing things like 'PLEASE TURN OVER'. We will check every page so we won't miss anything that's not on the first page.

(f) Don't write everything in small print on the first page only. You are allowed to write on later sides!

(g) We sometimes (not often) make mistakes! This year Question 3(b)(iii) had two possible answers, TRUE and FALSE. We fixed the answer schedule to allow for both answers. If you ever spot a mistake don't spend time worrying about it. Remember that every student will be disadvantaged by mistakes and we account for them when marking.