Department of Mathematics and Statistics
UNIVERSITY of OTAGO
Te Whare Wānanga o Otāgo
NEW ZEALAND

Junior Mathematics Competition 2018

Questions

Instructions to Candidates

You have **one** hour to answer **six** questions out of **eight**. The set of questions you answer is determined by your year level:

Question 1: 10 marks. Year 9 and below only.
Question 2: 10 marks. Year 10 and below only.
Question 3 to Question 6: 20 marks each. All students.
Question 7: 10 marks. Years 10 and 11 only.
Question 8: 10 marks. Year 11 only.

If you answer an incorrect question for your year level it will not be marked.

These questions are designed to test your ability to analyse a problem and express a solution clearly and accurately.

Please read the following Instructions carefully before you begin.

1. Do as much as you can. You are not expected to complete the entire paper. In the past, full answers to three full (20 mark) questions have represented an excellent effort.
2. You must explain your reasoning as clearly as possible, with a careful statement of the main points in the argument or the main steps in the calculation. Generally, even a correct answer without any explanation will not receive more than half credit. Likewise, clear and complete solutions to three full problems will generally gain more credit than sketchy work on four.
3. Credit will be given for partial solutions and evidence of a serious attempt to tackle a problem.
4. Textbooks are NOT allowed. Calculators may be used and students who do not have one may be disadvantaged. Otherwise normal examination conditions apply.
5. We recommend black or blue pens. Dark pencil is acceptable if you have nothing else. Do NOT use red or green pens, or light pencil that we cannot read.
6. We will penalise inappropriate rounding and incorrect or absent units.

Definitions

1. A prime number has exactly two factors, 1 and itself. By this definition 1 is not a prime number. The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.
2. A perfect square is an integer that is the square of an integer. The first ten perfect squares are 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100.

DO NOT TURN OVER UNTIL TOLD TO DO SO.

Question 1: 10 marks (Years 9 and below only)

(a) A rectangular wall measures 4.5 m by 3 m. Show that the area of the wall is 13.5 m^2.

(b) Suppose the area of a roll of wallpaper has dimensions 5 m by 0.5 m.

  (i) What is the area covered by one roll of wallpaper?
  (ii) How many 'whole' rolls of wallpaper would you need to cover the wall in part (a)? Your answer should not have a fraction or decimal component in it.

(c) If a window covers 2.7 m^2 on the wall, what percentage of the wall is covered by the window?

Question 2: 10 marks (Years 10 and below only)

A store sells refrigerators with a **recommended** price of $1600. However they know that they cannot sell any at this price so they offer them at a 'special' price of $1200.

(a) What is the percentage discount of $1200 compared to $1600?

(b) In a Boxing Day Sale they offer the fridge at a special '20% saving' i.e. 20% off the **recommended** price of $1600.

  (i) What is the special Boxing Day 'Sale' price?
  (ii) Does this Sale Price represent a saving on the 'special' price or an increase? You don't have to show working.

(c) They offer a 'further 20%' off the **Sale** Price if a customer pays cash. What percentage saving do these two percentages combined represent?

(d) Another store usually makes 400% profit on items they sell. For example an item costing the store $20 would sell for $100. In a sale they offer the $100 item for 50% off the usual price. What percentage profit on the item do they still make?

Question 3: 20 marks (All Years)

(a) Harry and Meghan plan to bake muffins for afternoon tea. However, the original recipe serves five people. They will have to adjust the amount of ingredients they need to serve the two of them.

  (i) The original recipe calls for 100 g of butter. How much butter is needed for two people?
  (ii) The original recipe calls for 15/16 of a cup of blueberries. What fraction of a cup do they need for two people? Briefly show your working.
  (iii) Harry and Meghan work out that they now need two cups of flour. How many cups of flour did the original recipe call for?
  (iv) They work out that for two people they need half a cup of plain yoghurt. How much plain yoghurt did the original recipe call for (i.e. for five people)?

(b) A good rule of thumb for converting ounces (oz) to grams (g) (although it is not perfect mathematically) is to use the formula 1 oz = 30 g. Use this to answer the following true or false questions (you don't need to show working):

  (i) 2 g = 60 oz.
  (ii) 1.6 oz = 48 g.
  (iii) 1 kg = 33 oz (to the nearest whole number).

(c) There are 16 ounces (oz) in a pound. Remember that we are using the approximation 1 oz = 30 g.

  (i) How many grams are there in 1 pound?
  (ii) How many pounds are there in a 6 kg of flour?

Question 4: 20 marks (All Years)

The ancient Egyptians only used two types of fractions, 2/3 and the unit fractions (fractions with 1 as the numerator), 1/2, 1/3, 1/4, 1/5 etc. They could write any fraction as additions of unit fractions:

e.g. 3/4 = 1/2 + 1/4 and 4/5 = 1/2 + 1/4 + 1/20 + 1/80.

(a) True or false? (You don't need to show working.)

(i) 2/3 = 1/2 + 1/6

(ii) 1/2 × 1/6 = 1/2 + 1/6

(iii) 2/3 of 1/5 = 1/10 + 1/30

(iv) $\frac{1/5}{1/2}$ = 1/10

(b) (i) Write 3/24 as a unit fraction.

(ii) Write 3/5 as the sum of exactly two **different** unit fractions.

(c) Divide 11 loaves of bread equally between 12 people, so that each person receives the same amount. Then give your answer as the sum of three **different** unit fractions.

(d) Consider the fraction 13/11. Write this as the sum of 2/3 and three unit fractions. In this question your unit fractions do not have to be all different.

Question 5: 20 marks (All Years)

A factorial is one of the 'building blocks' for many areas of Mathematics. For any whole number n, factorial n (denoted by $n!$) is defined as

$$n! = n \times (n - 1) \times (n - 2) \ldots \times 3 \times 2 \times 1.$$

For example, the number 5! = 5 × 4 × 3 × 2 × 1 = 120.

(a) Find the values of the following:

(i) 4!

(ii) (3 + 3)!

(iii) 10! / 8!

(iv) 16!/(14! × 2!)

(b) Write the following expressions as simply as possible:

(i) $n!/(n - 1)!$

(ii) $n!/[(n - 2)! \times 2!]$

A rowing coach has to select 2 rowers for a pairs competition out of 5 contenders. She decides to select them at random. This can be done in 10 ways. If the 5 rowers are Aroha, Bao, Charlotte, Debbie, and Emere, then she can pick any pair from this set:

{AB, AC, AD, AE, BC, BD, BE, CD, CE, DE}

The number 10 can also be found using 5!/(3! × 2!).

(c) In how many ways can an athletics coach pick a team of 4 athletes at random from 7 choices?

(d) In 'Litto', you win a prize in the first division by correctly picking 4 different numbers out of 15. What are your chances of winning a first division prize if you have one entry of different numbers selected at random?

(turn over)

Question 6: 20 marks (All Years)

(a) Find a set of ten consecutive numbers between 100 and 199 (inclusive), such that:

- Exactly one number is a perfect square.
- Exactly one number is prime.
- The other eight numbers have at least one of 2, 3, or 5 as a factor.

Explain your reasoning for the set you give. (In other words state the perfect square, the prime, and show the other numbers have one of 2, 3, or 5 as a factor.)

(b) Is the set of numbers you found in (a) the only such set of numbers that can be found? If not, give another set of numbers with explanations. If so, explain why there are no other sets that satisfy the requirements.

Question 7: 10 marks (Years 10 and 11 only)

Wiremu, a farmer, decides to put some lambs for two hours in a pen, one side of which is a very long brick wall. He will make the other sides from a long length of fencing. He doesn't need fencing for the side made from the brick wall. He decides that the pen will be 36 m^2 in area.

(a) If he makes a rectangular pen 2 m by 18 m (with the 18 m side being parallel to the brick wall), how much fencing will he use?

(b) If he does decide to make a rectangular pen with each side being a whole number of metres, find the minimum amount of fencing that he uses. Show working.

(c) Write 36 m^2 in hectares.

Question 8: 10 marks (Year 11 only)

Wiremu decides to make the pen an equilateral triangle shape with all three sides being equal in length and one side along the long brick wall. He will need to fence the other two sides and he decides that the area of the pen will still be 36 m^2. Find how much fencing he will use.
(Note: the final answer is not a whole number. Give your answer to one decimal place.)

(END OF COMPETITION)