UNIVERSITY of OTAGO
Te Whare Wānanga o Otāgo
NEW ZEALAND

Department of Mathematics and Statistics

Junior Mathematics Competition 2016

Questions

TIME ALLOWED: ONE HOUR

Only Year 9 and below candidates may attempt QUESTION ONE

ALL candidates may attempt QUESTIONS TWO to FIVE

These questions are designed to test ability to analyse a problem and to express a solution clearly and accurately.

Please read the following instructions carefully before you begin:

(1) Do as much as you can. You are not expected to complete the entire paper. In the past, full answers to three questions have represented an excellent effort.

(2) You must explain your reasoning as clearly as possible, with a careful statement of the main points in the argument or the main steps in the calculation. Generally, even a correct answer without any explanation will not receive more than half credit. Likewise, clear and complete solutions to two problems will generally gain more credit than sketchy work on four.

(3) Credit will be given for partial solutions and evidence of a serious attempt to tackle a problem.

(4) Textbooks are NOT allowed. Calculators may be used and students who do not have one may be disadvantaged. Translation devices without computational capability are permitted provided any communications capability has been switched off. Otherwise normal examination conditions apply.

(5) Diagrams are a guide only and are not necessarily drawn to scale.

(6) We recommend black or blue pens. Dark pencil is acceptable if you have nothing else. Do NOT use red or green pens, or light pencil that we cannot read.

(7) We will penalise inappropriate rounding and incorrect or absent units.

(8) You do not lose marks for incorrect answers.

DO NOT TURN OVER UNTIL TOLD TO DO SO.

Question 1 (Year 9 and below only)

Finn wants to buy a car to replace his old one. He drives from his home to the car salesperson's yard.

(a) On the way to the salesperson he travels at an average velocity of 25 km/hr. The entire trip takes 30 minutes. How far is it (in kilometres) from his home to the car salesperson's yard?

(b) Finn cannot afford the $12 000 cash price for his new car. He is offered two alternatives for finance:

  (i) A deposit of $1500 and then nine equal monthly payments of $1300 each time.

  (ii) A 10% deposit of the cash price and then nine equal monthly payments of $1320 each time.

  Which is the better (cheaper) deal? Explain your reasoning.

(c) After he has bought the new car, he goes for a drive on the open road. He travels at an average velocity of 80 km/hr. What is this in m/sec (metres per second)?

(d) When you are driving on the open road, it is recommended that you stop and rest every 100 km and <u>also</u> every two hours. Finn goes for a trip the following weekend. Travelling at a velocity of 80 km/hr on a 420 km journey, how many times should he stop and rest before the end of his journey if he follows all of the recommendations? Show your reasoning. (Do not count the pause times in your calculations.)

(e) On his way home, he passes another car. Travelling at 98 km/h he pulls out to pass. It takes two and a half (2.5) minutes to completely pass the other car. How far has he travelled in that time? Give your answer to the nearest tenth of a kilometre.

Question 2 (All students)

2016 is a leap year, which has 366 days (as opposed to the normal 365 days). A leap year is defined as any year divisible by 4, unless it is divisible by 100 and not 400.
For example, 2000 (a multiple of 400), 2020, and 1840 are leap years, while 1969 (odd), 1766 (not a multiple of 4), and 1900 (a multiple of 100 but not 400) are not leap years.

(a) For each of the following statements, answer true or false:

  (i) 1956 is a leap year.

  (ii) 1800 is a leap year.

  (iii) 1986 is a leap year.

(b) In the period 1816 to 2016 (inclusive), how many leap years were there?

(c) What was the last leap year prior to 2016 (since 1600) which was also a multiple of 30?

(d) Calculate the average number of days any year contains under the present leap year system. Give your answers to four decimal places and show necessary working.

(e) For how many days did the Māori Queen Dame Te Atairangikaahu (23 July 1931 – 15 August 2006) live altogether?

Question 3 (All students)

2016 marks the thirtieth anniversary of the Junior Mathematics Competition which began in 1986. In this question, we revisit some old questions (with changes).

PART A

The number 10 can be made by using each of the numbers 1, 9, 8, 6 exactly once and any of the standard operations +, − , ×, ÷, √, which may be repeated as necessary or not used at all, and any number of brackets. For example, 10 = 9 + 8 – 6 – 1.

(a) Use the same method to construct each of the numbers

(i) 20

(ii) 30

(b) We can construct the number 10 using the numbers 2, 0, 1, 6 in the same way, since 10 = (6 - 1) × 2 + 0. Is it possible to construct 20 in the same manner? If it is, show your construction. If not, briefly explain why it is impossible.

PART B

The number 1996 has six factors: 1, 2, 4, 499, 998, and 1996.

(c) List the factors that 1996 shares with the numbers

(i) 1986

(ii) 2006

(iii) 2016

(d) How many factors does the number 2016 have? (Hint: the number 1996 can be written as $2^2 \times 499$, where 2 and 499 are primes. Write 2016 in the same way.)

PART C

The numbers 1986 and 2006 share the property that when divided by 5 the remainder is 1, and when divided by 4 the remainder is 2.

(e) Explain why there are no numbers between 1986 and 2006 with the above property.

(f) Find non-negative integers *a, b, p,* and *q* (with *a* and *b* greater than 1 and *a* not equal to *b*) such that for any member *y* of the set
{1986, 1996, 2006, 2016}
we have remainder *p* on division of *y* by *a*, and remainder *q* on division of *y* by *b*.
(The numbers *a, b, p,* and *q* are the same for any value of *y*.)

Question 4 (All students)

(a) If a pie weighs 1400 grams, how much will a slice of pie weigh if it is shared equally by weight between 4 people?

(b) Suppose one person takes a 500 gram slice from a 1400 gram pie. If the remaining pie is shared equally by weight between 3 people, how much will each slice of the pie weigh?

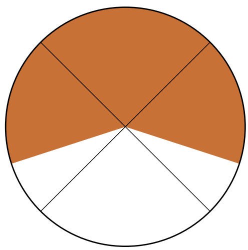

Figure 1 – not to scale

(c) Suppose three fifths of a pie sliced into four equal quarters is covered evenly with walnuts, as shown in Figure 1 (the portion in white has no walnuts, the darker brown portion does). If a slice of pie is partially covered with walnuts, what percentage of that slice is covered in walnuts?

(d) Suppose a 1400 gram pie is to be shared between 4 people, with three people taking an equivalent slice of pie by weight, and the fourth taking a slice of pie half the weight of the other slices. How much does each slice weigh?

(Question 4 continued on next page)

Question 4 (All students) continued from previous page

(e) A circular cake dish with a total diameter of 25 cm has a circular hole 1/8 of the total diameter cut into the centre, into which no ingredients can fall. What is the total volume of a cake if it completely fills the dish to a height of 10 cm?

Question 5 (All students)

(a) (i) Inside a square of side length 10cm is drawn another square, such that that the vertices of the sides of the smaller square touch the midpoints of the sides of the larger square (see Figure 2). Find the area of the smaller square.

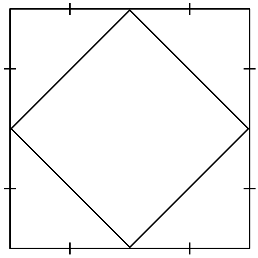

Figure 2 – not to scale

(ii) An even smaller square is drawn inside the second square on the same principles – the vertices of the third square touch the midpoints of the sides of the second square (see Figure 3). Find the area of the third square.

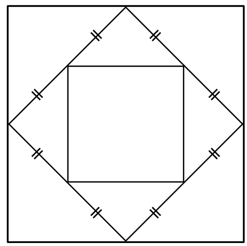

Figure 3 – not to scale

(iii) This process is repeated as long as possible. How many times altogether (i.e. starting at the beginning) do you need to carry out the process until you reach a square with area less than 1 cm^2?

(b) A square is drawn inside another square of side length 10cm so that the radius of the larger square is divided in the ratio 3:1 (see Figure 4). Find the area of the smaller square.

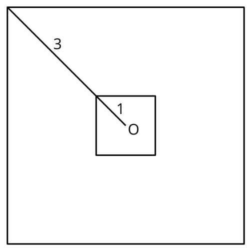

Figure 4 – not to scale

(c) A square is drawn inside another square of side length 10cm so that the vertices of the smaller square divide the sides of the larger square in the ratio of 3:1 (see Figure 5). Find the area of the smaller square.

(d) An octagon is drawn inside a square of side length 10cm. Four sides of the octagon meet the sides of the square and divide the sides of the square into thirds (see Figure 6). Find the area of the octagon.

(e) A square is drawn alongside another square of side length 10cm such that a vertex of the smaller square lies at the centre O of the larger square. The centre of the smaller square lies on the right hand edge of the first square, one quarter of the way up the edge (see Figure 7).

What is the area of the <u>overlap</u> between the squares?

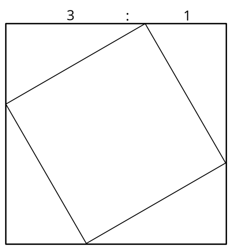

Figure 5 – not to scale

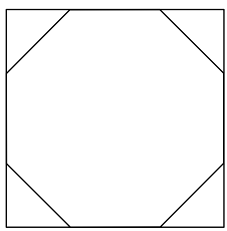

Figure 6 – not to scale

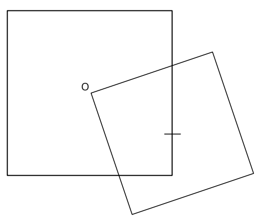

Figure 7 – not to scale

(END OF COMPETITION)